1. Evaluate the limit of the sequence \( \lim_{k \to \infty} \left( 1 - \frac{3}{k} \right)^{1/k} \).

**ANSWER:** 1

2. Evaluate the limit of the sequence \( \lim_{k \to \infty} \left( \sqrt{k+1} - \sqrt{k} \right) \sqrt{k} \).

**ANSWER:** 1/2

3. Decide whether the geometric series converges or diverges. If it converges find the sum \( \sum_{k=1}^{\infty} \frac{(-2)^k}{3^k} \).

**ANSWER:** converges \(-\frac{2}{5}\)

4. Find the sum of the telescoping series \( \sum_{k=2}^{\infty} \frac{1}{(k+1)(k-1)} \).

**ANSWER:** converges \(3/4\)

5. Evaluate the integral \( \int_{1}^{\infty} \frac{dx}{x \ln(x)} \). Determine convergence or divergence of \( \sum_{k=2}^{\infty} \frac{1}{k \ln(k)} \).

**ANSWER:** \(\infty\), Series diverges by integral test

In problems (6) - (8) decide which of the following are true for the given series: Converges Absolutely; Converges Conditionally; Diverges. (Note: You must give reasons and show all work).

6. Apply the limit comparison test to \( \sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^2 + 1} \) with \( b_k = \) and \( L = \lim_{k \to \infty} \frac{a_k}{b_k} = \)

**ANSWER:** \(b_k = \frac{1}{k^{3/2}}\), \(L = 1\), converges absolutely by limit comparison and p-test

7. Apply the ratio test to \( \sum_{k=1}^{\infty} \frac{k^2 2^k}{k!} \). \( L = \)

**ANSWER:** Ratio test \(L = 0\) so series converges absolutely.

8. Apply root test to \( \sum_{k=1}^{\infty} k \left( \frac{3}{4} \right)^k \). \( L = \)

**ANSWER:** Root test \(L = 3/4\) so series converges absolutely.

9. Find all values of \( x \) for which the series converges. Make sure to test the endpoints, \( \sum_{k=1}^{\infty} \frac{x^k}{k3^k} \).

**ANSWER:** Ratio test \(L = \frac{|x|}{3}\) so series converges absolutely for \(|x| < 3\). At \(x = 3\) the series diverges to infinity and at \(x = -3\) the series diverges.

10. Find the Maclaurin series for \( f(x) = \frac{x}{1-x^2} \) and give the radius of convergence.

**ANSWER:** Since \( \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k \) so we have \( \frac{x}{1-x^2} = \sum_{k=0}^{\infty} x^{2k+1} \) with infinite radius of \( R = 1 \). At \( x = \pm 1 \) series diverges.