1. Find the radius of convergence of the power series \[ \sum_{n=0}^{\infty} \left( \frac{n}{n^2 + 1} \right) x^n \]

\[
\lim_{n \to \infty} \left| \frac{a_n}{a_{n+1}} \right| = \lim_{n \to \infty} \left( \frac{(n+1)^2 + 1}{n^2 + 1} \right) \left( \frac{n}{(n+1)} \right) = 1
\]

2. Find the radius of convergence of the power series \[ \sum_{n=0}^{\infty} \left( \frac{3}{2} \right)^n x^{2n} \]

First set \( t = x^2 \) and find radius of convergence in \( t \)

\[
\lim_{n \to \infty} \frac{1}{\left| a_n \right|^{1/n}} = \lim_{n \to \infty} \frac{1}{((3/2)^n)^{1/n}} = \frac{1}{3/2} = \frac{2}{3}
\]

Now we have convergence for \( |x^2| = |t| < 2/3 \) so the series in \( x \) converges for \( |x| < \sqrt{2/3} \)

3. Find the first four terms of \( y(x) = \sum_{n=0}^{\infty} a_n x^n \) in terms of \( a_0 \) for \( y' + 2xy = 0 \).

\[
y'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1} \text{ so that } \sum_{n=1}^{\infty} n a_n x^{n-1} + 2 \sum_{n=0}^{\infty} a_n x^{n+1} = 0 \text{ and }
\]

\[
0 = \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n + 2 \sum_{n=1}^{\infty} a_{n-1} x^n = a_1 x^1 + \sum_{n=1}^{\infty} [(n+1)a_{n+1} + 2a_{n-1}] x^n
\]

Thus we have \( a_1 = 0 \) which implies that also \( a_3 = a_5 = \cdots = 0 \) and for \( n = 1, 3, 5, \cdots \) we get

\[
a_{n+1} = \frac{-2a_{n-1}}{(n+1)}
\]

so with \( a_0 \) arbitrary we have the next three terms \( a_1 = 0, a_2 = -a_0 \) and \( a_3 = 0 \).