1. Consider the problem

\[ y'' + \epsilon y' + y = 0, \quad y(0) = 1, \quad y'(0) = 0. \quad (*) \]

(a) Find the exact solution to this initial value problem.
(b) Obtain a two term regular asymptotic expansion for (*)
(c) Compare graphically your answer for \( \epsilon = .25 \) and \( \epsilon = .1 \) with the exact answer for \( 0 \leq x \leq 2 \).

2. Obtain a two term regular expansion for \( y'' + 2y = e^{\epsilon x}, \ y(0) = y(1) = 0 \).

3. Obtain a two term regular expansion for \( (1 + \epsilon x^2)y'' + y = x^2, \ y(0) = \epsilon, \ y(1) = 1 \).

4. Find a two term expansion for \( y'' = \sin(x)y, \ y(0) = 1, \ y'(0) = 1 \) using the method of successive integration from Example 5.4.

5. Find a regular expansion for the system \( \begin{cases} \dot{x} = x - 2y + \epsilon xy \\ \dot{y} = x - 3y - \epsilon xy \end{cases} \) Is the expansion valid for all \( t \geq 0 \)? Give a reason.

6. Use the method of strained variables to obtain a two term expansion for

\[ (x + \epsilon y) \frac{dy}{dx} + y = 0, \quad y(1) = 1. \]

Also find the exact solution and compare graphically your results and the exact solution on the interval \([0, 2]\) for \( \epsilon = .1 \).

7. Use the method of averaging to find an approximate periodic solution to the Van der Pol oscillator

\[ \dot{y} + y + \epsilon(y^2 - 1) \dot{y} = 0, \]

i.e., find an approximation \( y(t) \sim a(t) \cos(t + \theta(t)), \quad a(0) = a_0, \quad \Theta(0) = \Theta_0 \).

8. Find a uniform asymptotic approximation to the boundary layer problem, i.e., find an inner, outer and matched solution.

\[ \epsilon y'' + (1 + \epsilon)y' + y = 0, \quad y(0) = 0, \quad y(1) = 1. \]

Also compute the exact solution and graphically compare your answers for \( \epsilon = .1 \) and \( \epsilon = .025 \).

9. Find a uniform asymptotic approximation to the boundary layer problem, i.e., find an inner, outer and matched solution.

\[ \epsilon y'' + 2y' + e^y = 0, \quad y(0) = 0, \quad y(1) = 0 \]