

# TOPOLOGY DOCTORAL PRELIMINARY EXAMINATION TOPICS

Questions for the topology doctoral preliminary examination will come from the following topics. Students are expected to demonstrate sufficient depth and breadth of knowledge in these areas by constructing examples or counterexamples and by proving theorems involving these topics. Problems may also address behavior of these properties under continuous functions, products, quotients and other structures or characterizations of these properties. Some problems may involve combinations of several of the topics listed below or implications between the various properties. The specific nature of the exam and its grading will be determined by the specialists in topology on the mathematics faculty.

1. **Set Theory:** cardinality, ordinals, functions, axiom of choice and equivalences
2. **Regular, completely regular and normal spaces, paracompactness**
3. **Covering Properties:** compact spaces, Lindelof spaces, variations on compactness (local, sequential, countable, etc.), separable spaces and dense sets, first countable and second countable spaces.
4. **Connectedness, locally connected or arcwise connected spaces, continua**
5. **Extensions of spaces:** embeddings, compactifications, completions
6. **Basic Constructions:** product spaces, quotient spaces and decompositions, ordered spaces and subspaces of ordered spaces, inverse systems, subspaces, finer and coarser topologies, sequences, weak topology
7. **Maps:** continuity, function spaces, extensions, retracts, special types of maps (open, closed, perfect, monotone, etc.)
8. **Metrizability:** generalized metric structure, metrization theorems, complete metric spaces, extension theorems
9. **Topology manifolds, topology of the plane**
10. **Homotopy Theory:** fundamental groups, covering spaces, homotopy equivalences, computation of homotopy groups, knots and knot complements
11. **Homology Theory:** simplicial structures, axioms for homology theories, relative and local homologies
12. **Topological groups, transformation groups, topological dynamics**