Prelim Topics For ODE/ PDE Exam

I. Ordinary Differential Equations

1. Examples of Explicit Solution Techniques
2. Initial Value Problems
   (a) Existence and Uniqueness of Solutions
   (b) Continuation and Maximal Intervals of Existence
   (c) Dependence on Data
   (d) Applications of Gronwall type inequalities
3. Linear Differential Equations
   (a) N^{th} Order Linear Equations
   (b) Linear Systems with Constant Coefficients
   (c) Matrix Algebra (including Marix Exponentials and Canonical Forms)
   (d) General Solutions and Vector Space Concepts
   (e) Wronskian
   (f) Abel Formula
   (g) Fundamental Set, Fundamental Matrix, Fundamental Solution
   (h) Variation of Parameters Formula for Nonhomogeneous Equations
4. Stability Theory
   (a) Stability of Linear Systems with Constant Coefficients
   (b) Routh-Hurwitz Criteria
   (c) Stability of Linear Systems with Variable Coefficients
   (d) Autonomous Systems and the Phase Plane
   (e) Stability of Nonlinear Systems
   (f) Stability by Linearization
   (g) Lyapunov Functions and Lyapunov Theorems
5. Boundary Value Problems for Second Order Equations
   (a) LaGrange's identity, Green's Formula
   (b) Canonical Forms, Self-Adjoint Boundary Conditions
   (c) Oscillation, Separation and Comparison Theory
   (d) Sturm-Liouville Boundary Value Problems
   (e) Green's Functions
   (f) Eigenvalues and Eigenfunction Expansions

II. Partial Differential Equations

1. Linear and Quasilinear equations of First Order
   (a) Single First Order Equations.
   (b) Vector Fields and Integral Curves and Surfaces.
   (c) Quasilinear Equations in $\mathbb{R}^2$: shocks, conservation laws, etc.
   (d) Characteristics for Second Order Equations and Higher Order Equations

(e) The Cauchy Problem for Higher Order Equations and Well Posedness
(f) Holmgren Uniqueness Theory
(g) Smoothness of Solutions: Cauchy-Kovalevski Theorem, Classical Lewy example equation with no C^1 solution

2. Canonical Forms
   (a) Reduction to Canonical Forms
   (b) Classification of **2**nd Order Equations in Two Variables: Hyperbolic, Elliptic Parabolic.
3. Hyperbolic Equations
   (a) The One Dimensional Wave Equation
   (b) D'Alembert's Formula
   (c) Energy Methods
   (d) Domain of Dependence, Range of Influence
   (e) Forward and Backward Characteristic Cones
   (f) The Cauchy Problem in $\mathbb{R}^n,\ n = 2, 3$
   (g) Spherical Means and the Solution in $\mathbb{R}^3$
   (h) The method of Descent and the Solution in $\mathbb{R}^2$
   (i) Duhamel's Principle
   (j) Wave Equation in Bounded Domains
   (k) Conservation of Energy
   (l) Examples of Boundary Value Problems
   (m) Separation of Variables and Eigenfunction Expansions
4. Elliptic Equations
   (a) Laplace and Poisson's Equations
   (b) Harmonic Functions
   (c) Boundary Value Problems
   (d) Green's Identities and Uniqueness
   (e) Fundamental Solutions and Green's Functions
   (f) The Maximum Principle and its Consequences
   (g) The Mean Value Theorem
   (h) Separation of Variables and Eigenfunction Expansions
5. Parabolic Equations
   (a) Some Results from Functional Analysis
      i. The classical L^p function spaces
      ii. Hölder's inequality
      iii. Young' Inequality
      iv. Convolutions
      v. Mollifiers and Approximate Identity
      vi. Schwartz class $\mathcal{D}$ of smooth compactly supported functions
      vii. $\mathcal{S}$ of rapidly decreasing functions
      viii. Fourier transform
      ix. Riemann-Lebesque Lemma
      x. Plancherel Formula

(b) The Heat Equation in $\mathbb{R}^n$
(c) The Heat Equation in Bounded Domains
(d) Maximum Principle and Uniqueness Theorems
(e) Distributions
(f) Fundamental Solutions
(g) Separation of Variables and Eigenfunction Expansions

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