A Very Brief Introduction to Matlab

1. In matlab every object is a complex matrix in which real entries are displayed as real and integer as integer.

2. For numbers (i.e., 1 × 1 matrices) the syntax for arithmetic is standard

   \[ \begin{align*}
   1+2 \\
   3*5 \\
   5/8 \\
   3^4 \\
   \end{align*} \]

   For matrices of larger dimensions there are certain compatibility and other conditions that need to be met.

3. Numbers (and matrices) can be assigned to variables using the equal sign, e.g.,

   \[ \begin{align*}
   a &= 3 \\
   b &= 4 \\
   a+b \\
   \end{align*} \]

4. There are several ways to enter a matrix into Matlab's workspace:

   (a) You can type in the elements

       \[ A = \begin{bmatrix}
       2 & 4 & 5 \\
       2 & 6 & 3 \\
       -1 & 6 & 2 \\
       \end{bmatrix} \]

       builds a 3 × 3 matrix. The entries are typed in rows (elements separated by a space or a comma) with a semicolon used to declare the beginning of a new row.

   (b) You could also generate a random matrix using the commands \( \text{rand}(n) \) or \( \text{rand}(n,m) \) to generate a random \( n \) by \( n \) or \( n \) by \( m \) matrix whose elements are normally distributed in 0 to 1, e.g., the command

       \[ a = \text{fix}(10*\text{rand}(5)) \]

       generates a 5 × 5 matrix with integer entries.

   (c) Using a loop to construct the entries of a matrix. The following syntax builds the 5 × 5 matrix

       \[ A = \begin{bmatrix}
       1 & 1/2 & 1/3 & 1/4 & 1/5 \\
       1/2 & 1/3 & 1/4 & 1/5 & 1/6 \\
       1/3 & 1/4 & 1/5 & 1/6 & 1/7 \\
       1/4 & 1/5 & 1/6 & 1/7 & 1/8 \\
       1/5 & 1/6 & 1/7 & 1/8 & 1/9 \\
       \end{bmatrix} \]
for n=1:5
    for m=1:5
        A(n,m) = 1/(n+m-1);
    end
end

5. For example, in matlab we could enter a row vector with three entries as

\[
x=[10.1 \ 20.2 \ 30.3]
\]

or a column vector as

\[
x=[10.1;20.2;30.3]
\]

6. If you enter these expression in Matlab the result will define x to be the appropriate right hand side and will then display the result on the screen. If you don’t want to display the result simple end the line with a “semi-colon.” For example,

\[
x=[10.1 \ 20.2 \ 30.3]
\]

will set x equal to the right hands side but suppress printing the output.

7. Once x has been defined it can be displayed any time by simply typing x and return.

8. To transform a row vector of real numbers x into a column vector of real numbers y simply compute the transpose which is given by a “single quote” as in

\[
y=x'
\]

9. A vector of equispaced elements can be generated using the general format:

\[
\{\text{beginning number}\} : \{\text{step increment}\} : \{\text{last number}\}
\]

If the step is 1 then it can be omitted

\[
\{\text{beginning number}\} : \{\text{last number}\}
\]

so that \(x = 1 : 6\) is the vector \(x=[1 \ 2 \ 3 \ 4 \ 5 \ 6]\). You can also use a negative incremental step size, if the first number is smaller than the last. For example
x=10:-1:1

builds the vector [10 9 8 7 6 5 4 3 2 1].

10. It is important to be able to find on-line help: You should try

(a) help who
(b) help whos
(c) help clear
(d) help zeros
(e) help ones
(f) help eye
(g) help ;
(h) help []
(i) help linspace
(j) help plot
(k) help length
(l) help size

11. Consider an $n \times m$ matrix $A$:

(a) to learn the size (i.e, the number of rows and columns) type $[n,m]=\text{size}(A)$

(b) If $A = [a_{ij}]$ and $B = [b_{ij}]$ are $n \times m$ matrices and $C = [c_{ij}]$ is an $m \times p$ matrix, then we have the following matrix arithmetic operations:

i. $A + B = [a_{ij} + b_{ij}]$ and $A - B = [a_{ij} - b_{ij}]$

ii. $A \ast C = D$ where $D$ is a $n \times p$ matrix with entries $d_{ij} = \sum_{k=1}^{m} a_{ik}c_{kj}$. For matrix multiplication the number of columns of the first matrix must be the same as the number of rows of the second.

iii. For a number $\alpha$, the scalar product $\alpha A = [\alpha a_{ij}]$

For example

$A=[1 \ 2 \ 5 ;-2 \ 1 \ 4]$

$B=[4 \ 2 \ 0 ;4 \ 2 \ -7]$

$C=[3 \ 6 ;-2 \ 1 ;-4 \ 2]$
The usual rules of positive integer exponents applies for square matrices, \( A^2 = A \times A, \ A^3 = A \times A \times A, \)
\[
A^r = \underbrace{A \times A \times \cdots \times A}_{r}.
\]

\[
A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}
\]

\( A^2, \ A^3 \)

In general, division of matrices makes no sense. But for nonsingular square matrices is it is possible to make an interpretation of matrix division. If \( A \) is nonsingular, then it has an inverse, i.e., a matrix \( A^{-1} \) satisfying \( A \times A^{-1} = A^{-1} \times A = I \) where \( I \) is the identity matrix with ones on the main diagonal and zeros elsewhere. The identity matrix plays the same role as the number 1 does for multiplication \( A \times I = I \times A = A \). In Matlab the identity matrix is given by \( \text{eye}(n) \) where \( n \) is an integer. If \( A \) is nonsingular, then the inverse in Matlab is given by \( \text{inv}(A) \) or \( A^{-1} \). In this case we can think of division as \( B \times A^{-1} \) just as we do with numbers \( b \div a = b \times a^{-1} \) for numbers with \( a \neq 0 \).

If \( A \) has an inverse then you can compute \( C = A^{-r} \).

\[
A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}
\]

\( A^{-2}, \ A^{-3} \)

A matrix has an inverse if and only if its determinant is not zero. Recall the determinant for a square matrix is a number. You can find the definition of the number in most college algebra books. In Matlab it is easy to compute determinants using the command \( \text{det} \).

\[
A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}
\]

\( d = \text{det}(A) \)

Recall for a 2 \( \times \) 2 matrix

\[
\text{det} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11}a_{22} - a_{12}a_{21}.
\]
The problem of determining when a square matrix has an inverse is not easy to answer. The answer can be cast as a definition: We say a matrix is nonsingular if and only if it has an inverse.

There are several other characterizations of nonsingular given below. We will consider these properties with two examples

\[ A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} \]

\[ B = \begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix} \]

A matrix \( A \) is nonsingular if and only if any one of the following hold:

i. \( \det A \neq 0 \)

\( \det(A) \)

\( \det(B) \)

ii. The row reduced echelon form of \( A \) is the identity

\( \text{rref}(A) \)

\( \text{rref}(B) \)

iii. \( A \) has an inverse

\( \text{inv}(A) \)

\( \text{inv}(B) \)

iv. The only solution of the equation \( Ax = 0 \) is \( x = 0 \), i.e., the null space is the zero vector. The Matlab command “null” computes a basis for the null space. Note It does not list the zero vector.

\( \text{null}(A) \)

\( \text{null}(B) \)

v. The matrix \( A \) has full rank. If \( A \) is \( n \times n \) then the rank of \( A \) is \( n \).

\( \text{rank}(A) \)

\( \text{rank}(B) \)

(i) the \( i \)th row is \( A(i,:) \)

(j) the \( j \)th column is \( A(:,j) \)

(k) if \( 1 \leq i \leq j \leq n \) and \( 1 \leq p \leq q \leq m \), the statement \( B=A(i:j,p:q) \) gives the matrix

\[
B = \begin{bmatrix}
    a_{ip} & \cdots & a_{iq} \\
    \vdots & \vdots & \vdots \\
    a_{jp} & \cdots & a_{jq}
\end{bmatrix}
\]

(l) the statement \( B=A(i,p:q) \) gives the matrix

\[
B = \begin{bmatrix}
    a_{ip} & \cdots & a_{iq}
\end{bmatrix}
\]
(m) the statement \( B=A(i:j,p) \) gives the matrix \( B = \begin{bmatrix} a_{ip} \\ \vdots \\ a_{jp} \end{bmatrix} \)

(n) the statement \( B=A(i:j,:) \) gives the matrix \( B = \begin{bmatrix} a_{i1} & \cdots & a_{im} \\ \vdots & \cdot & \vdots \\ a_{j1} & \cdots & a_{jm} \end{bmatrix} \)

(o) the statement \( B=A(:,p:q) \) gives the matrix \( B = \begin{bmatrix} a_{1p} & \cdots & a_{1q} \\ \vdots & \cdot & \vdots \\ a_{np} & \cdots & a_{nq} \end{bmatrix} \)

12. Hadamard (or “dot”) operations. The Hadamard multiply \( .* \), divide \( ./ \) and exponentiate \( .^\) are very useful and powerful syntax.

- \( C=A.*B \) has entries \( c(i,j)=a(i,j)b(i,j) \)
- \( C=A./B \) has entries \( c(i,j)=a(i,j)/b(i,j) \)
- \( C=A./B \) has entries \( c(i,j)=b(i,j)/a(i,j) \)
- \( C=A.^B \) has entries \( c(i,j)=a(i,j)^{b(i,j)} \)
- \( C=A.^2 \) has entries \( c(i,j)=a(i,j)^r \) , \( r \) a number.
- \( C=r.^A \) has entries \( c(i,j)=r^{a(i,j)} \)

For example, let

\[
A=\begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}
\]
\[
B=\begin{bmatrix} -4 & 1 \\ 3 & 2 \end{bmatrix}
\]

\[
A.*B
\]

\[
A./B
\]

\[
A.^B
\]

\[
A.^2
\]

\[
2.^B
\]

This is very useful in evaluating vectorially a function that has products of elementary functions (for example, polynomials of a matrix).

13. Suppose for example that you wanted to plot the function \( y = x \sin(x^2) \) on the interval \([-4, 4]\).

\[
x=\text{-}4:.01:\text{4};
\]

\[
y=x.*\sin(x.^2);
\]

\[
\text{plot}(x,y)
\]